5.3 Notes and Examples

Name: Block: Seat:

5.3: Derivatives of Inverse Functions

1. Warm Up Questions
   (a) If $g(2) = 3$ and $g'(2) = -4$, and $f(x) = x^2 \ln(g(x))$, find $f'(2)$

   (b) Find the derivative of $y = x^{2x+3}$

   (c) If $y = \dfrac{x}{\ln x}$ then $y' = \dfrac{\ln x - 1}{(\ln x)^2}$, and $y'' = \dfrac{2 - \ln x}{x(\ln x)^3}$. Find and justify any relative extrema.

2. Inverse functions (as seen in previous courses):
   (a) If $f(a) = b$ and always b, never something else, then f is a function. If we have the graph of f we can use the __________ line test to determine if f is a function.

   (b) If a function is "injective", that is ____________, it passes the ____________ line test, and we know its inverse is also a function.

   (c) x's and y's are ____________

   (d) Domains and ranges are ____________

   (e) graphs are symmetric over the line ____________

   (f) f and g are inverses if and only if $f(g(x)) =$ ____________ $= g(f(x))$

   (g) ________________ functions (always increasing or always decreasing) will _____________ have an inverse that is a function.

   (h) Notation: $f^{-1}(x)$ is the _______________ of f. The inverse of $g(x)$ is written _____________

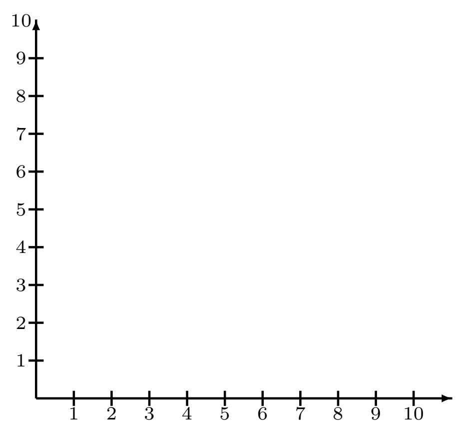
3. ALWAYS restrict the the domain of the inverse function to the range of the function.
   (a) Find $f^{-1}(x)$, the inverse of $f(x) = 6x + 2$

   (b) Find $g^{-1}(x)$, the inverse of $g(x) = \sqrt{x-5}$. Find the domain and range of $g^{-1}(x)$

And now the Calculus of inverse functions...

4. Consider $f(x) = \dfrac{1}{2}x^2 + 2$ on $[0, \infty)$ and $f^{-1}(x) = \sqrt{2x-4}$ on $[2, \infty)$
   (a) $f(4) =$

   (b) $f^{-1}(10) =$



   (c) $f'(4) =$

   (d) $(f^{-1})'(10) =$

   (e) Go to `https://www.desmos.com/calculator/znclra2xmj` and Press the big $\boxed{+}$ and select "$f(x)$ **expression**" to add the tangent line equations at these two points. (Use $y-b = m(x-a)$ form if you like). Notice anything interesting?

   (f) Go to `https://www.geogebra.org/m/JyWqdaZM` and type in $0.5x^2 + 2$, our function f, and move the $x(A)$ slider to 4. Move it to other values. What is the relationship between the slopes of f and f^{-1}?

5. At their coressponding points, the slopes of the tangent lines will be ____________ of each other.

$f(a) = b$	$f^{-1}(b) =$
$f'(a) = c$	$(f^{-1})'(b) =$

(a) Without finding the inverse function, find $(f^{-1})'(-3)$ for $f(x) = x^3 + 4x + 2$.
Steps
1. Find x so that $f(x) = -3$
2. fill in the top row
3. Find $f'(x)$
4. fill in the bottom row

(b) Without finding the inverse function, find $(f^{-1})'(1)$ for $f(x) = \sqrt{x^3 - 7}$.
make your own box to organize the info

(c) AP Style question: If $g(f(x)) = x$, $g(7) = 2$, and $g'(7) = 10$, then $f'(2)$ is
make your own box to organize the info

(d) AP Style question: If $g(f(x)) = x$, $g(9) = 3$, and $g'(9) = -4$, then $f'(3)$ is
make your own box to organize the info